Robustness Measures for Stochastic Parallel Machine Scheduling and the Train Unit Shunting Problem

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- Simulation (e.g. vdA, van Blokland, Hoogeveen, 2013):
  - Taking many samples from the processing time distributions.
  - Accurate estimation, but computationally expensive.
- **Robustness measures** (RMs): analytical functions that estimate robustness based on the schedule structure.
- Many RMs have been suggested, no consensus in which one works best

Our contribution

- Overview of robustness measures, including a few new ones
- Elaborate computational study to assess their quality
- Application of these measures for the Train Unit Shunting Problem

Stochastic parallel machine scheduling

- We consider stochastic processing times, where p_j is from a given distribution D_j.
- Construct a **baseline schedule**
  - Use **Local Search** to find baseline schedule
- Policy in schedule execution:
  - Ordering of jobs and machine assignment from base line schedule
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Jobs may not start on their planned start time and the makespan will not be as planned.

We want to create **robust** baseline schedules.

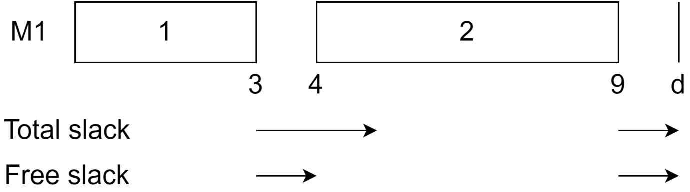
Robustness

Robustness: ability of a schedule to withstand disruptions

- **Quality robustness**: the objective function is unlikely to degrade under uncertainty.
  - e.g. schedule is likely to meet deadline
- **Solution robustness**: the schedule itself is unlikely to degrade under uncertainty.
  - e.g. jobs are likely to start on their planned start time.

Slack

- **Total slack** of a job (Leon, Wu, Storer, 1994):
  - The maximum amount of time that we can delay a job without exceeding the deadline.
- **Free slack** of a job (Al-Fawzam and Haouari, 2005):
  - The amount of time by which we can delay the job without delaying any other job in the schedule.



job	1	2
p_j	3	5
TS_j	2	1
FS_j	1	1

Slack based robustness measures

Sum:	$RM_1(S) = \sum_j TS_j^S$
	$RM_2(S) = \sum_j FS_j^S$
Minimum:	$RM_3(S) = \min_j\{TS_j^S\}$
	$RM_4(S) = \min_j\{FS_j^S / p_j\}$
	$RM_5 = \sum_j \min\{FS_j, \lambda p_j\}$
Counting:	$RM_6(S) = \sum_j \alpha_j$ where $\alpha_j = 1$ if $FS_j^S > 0$
Weighted sum:	$RM_7(S) = \sum_j FS_j^S \times p_j$
	$RM_8(S) = \sum_j FS_j^S \times NDP_j^S$
	$RM_9(S) = \sum_j FS_j^S \times NDS_j^S$
	$RM_{10}(S) = \sum_j FS_j^S \times p_j \times NDS_j^S$
Slack sufficiency:	$RM_{11}(S) = \sum_j \lvert\{i \mid i \in prec_j^S \cup \{j\}, FS_j^S \geq \lambda p_i\}\rvert$
	$RM_{12}(S) = \sum_j \lvert\{i \mid i \in prec_j^S \cup \{j\}, FS_j^S < \lambda p_i\}\rvert$
	New, cost of being non-robust, RM_{11} favors spreading prec over machines
Interval schedule	$RM_{13}(S) = \max \sum_j (l_j - e_j)$, $[e_j; l_j]$ execution interval for job j, LP
	$RM_{14}(S) = \max \min_j (l_j - e_j)$, $[e_j; l_j]$ execution interval for job j, LP

Robustness measures: Normal approximation

Approximation based on normal distribution (Passage, vdA, Hoogeveen,2016):

- Assumption that all job times are normally distributed.
- Compute **normal distributions** of start and completion times.
  - Start time: $S_j = \max\{PST_j, \max_{i \prec j} C_i\}$
  - Completion time: $C_j = S_j + D_j$
  - Use order imposed by precedence relations and machine sequence
  - Find the expected value and variance of the maximum of 2 normal distributions in every step (Nadarajah and Kotz, 2008)

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Resulting robustness measures

- $RM_{15}(S) = P(\max_j CT_j \leq d)$, probability of meeting target d
- $RM_{16}(S) = \sum_j P(ST_j \leq PST_j^S)$ (new), total probability of jobs starting on time

Measures based on predecessor slack (new)

- $RM_{17}(S) = \sum_j \alpha_j$
  - α_j is the fraction of direct predecessors with a free slack of at least λp_i (and 1 if there are no direct predecessors)
- $RM_{18}(S) = \sum_j ESD_j^S$
  - total estimated start time delay for the case in which job j has a delay of exactly λp_j

Simulation study: Evaluating robustness measures

For each instance (we did 24):

1. Determine a collection of 970 baseline schedule (with slack)
   - Basic local search and rules to assign random slacks
2. Determine **correlation** between RMs and “true” robustness by simulation.

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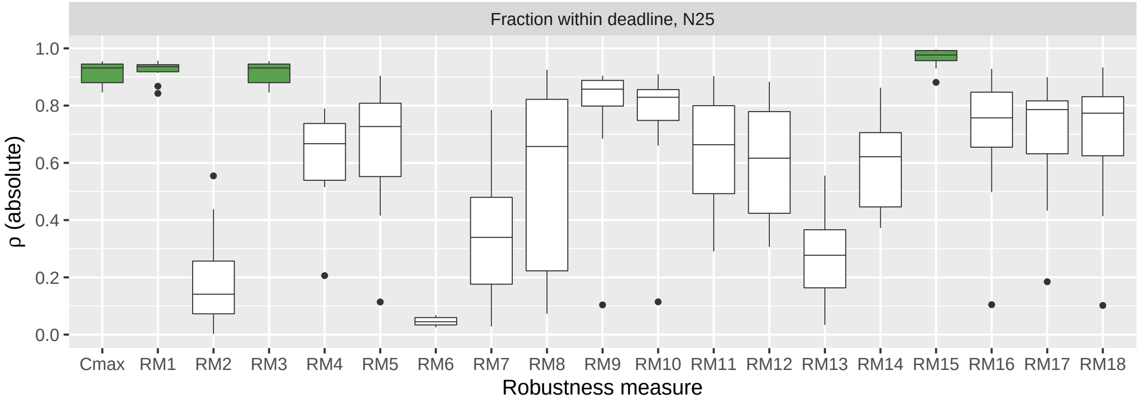
Determine correlation between RMs and "true" robustness.

- Simulation to accurately estimate true robustness (1000 runs).
  - Average makespan (quality robustness).
  - Fraction of runs within deadline (quality robustness).
  - Average fraction on time jobs (solution robustness).
  - Average sum of job delays (solution robustness).
- Report Spearman's Rank correlation coefficient ρ

Simulation study: Results quality robustness



Spearman's Rank correlation coefficient ρ (absolute) for fraction within deadline. Green highlight is mean ≥ 0.9

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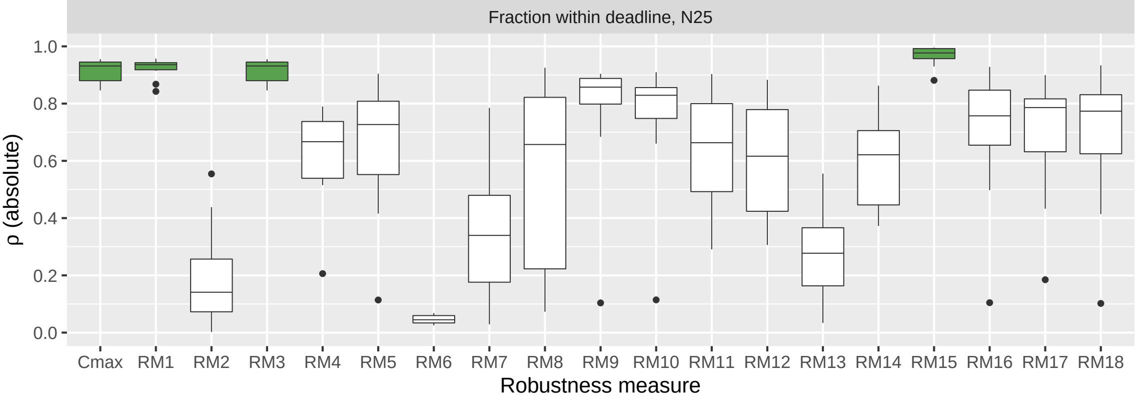

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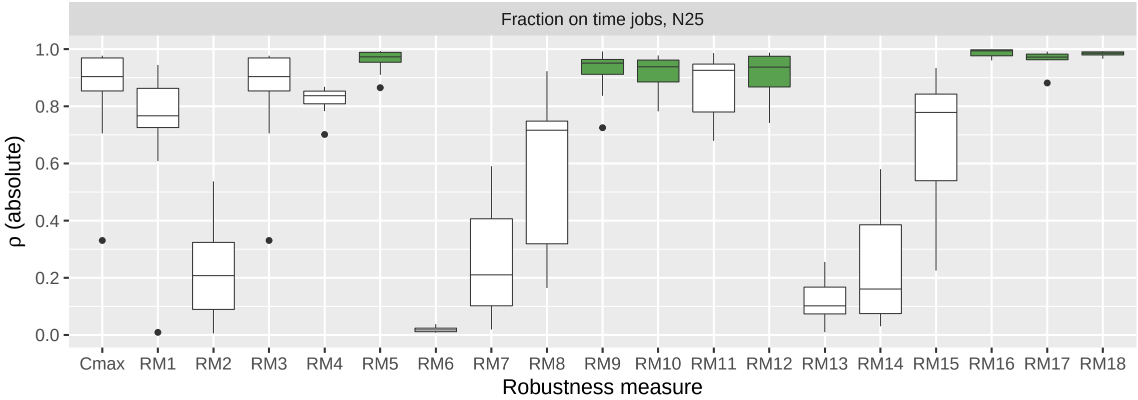
Sum of total slack $RM_1 = \sum_j TS_j$

Minimum total slack (related to $C_{\max}$) $RM_3 = \min_j TS_j$

Normal approximated probability within deadline $RM_{15} = P(\max_j C_j \leq d)$

Simulation study: Results solution robustness



Spearman's Rank correlation coefficient ρ (absolute) for fraction on time jobs using distribution N_{25}. Green highlight indicates mean ≥ 0.9.

Best RMs solution robustness

$RM_5 = \sum_j \min\{FS_j, \lambda p_j\}$	Where λp_j increase in processing time we want to survive
$RM_{16} = \sum_j P(S_j \leq PST_j)$	Normal approximated probability of jobs starting on time
$RM_{17}(S) = \sum_j \alpha_j$	Where α_j is the fraction of direct predecessors with a free slack of at least λp_i (and 1 if there are no direct predecessor)
$RM_{18}(S) = \sum_j ESD_j^S$	Total estimated start time delay for the case in which job j has a delay of exactly λp_j
$RM_9 = \sum_j FS_j \times NDS_j$	Sum of free slack scaled by number of direct successors
$RM_{10} = \sum_j FS_j \times NDS_j \times p_j$	Sum of free slack scaled by number of direct successors and processing time

Conclusion from simulation study

- Normal approximation works well
- Total slack good measure for quality robustness
  - Minimum total slack (RM_3) is slack on critical path
  - Equivalent to makespan for given machine assignment and order
- Including λp_i as delay that we want to survive seems to work
- Free slack works best if weighted by number of direct successors or maximized at 'survivable' delay λp_i.

The Train Unit Shunting Problem (TUSP)

- Making a schedule for a shunting yard using local search (van den Broek, Hoogeveen, van den Akker, Huisman 2018).
  - Matching incoming to outgoing trains.
  - Combining and splitting train compositions.
  - Parking trains.
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Robustness measures for the TUSP

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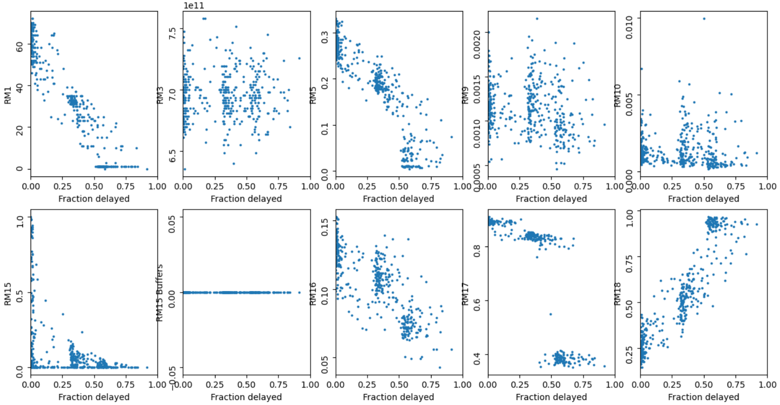
- Algorithm by Van den Broek et al. Gives us a full schedule including start times of jobs, and precedence constraints between different jobs.
- Use this schedule to calculate the robustness measures from earlier.



Local Search study

- Use measures with best correlation result as objective in Local search for TUSP.
  - Single Robustness measure.
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  - planned start time.
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- Measures based on normal approximation seem most effective (RM_{15}, RM_{16}).
- Effectiveness of measures dependent on execution policy.
- Solution robust measures can also increase the quality robustness.

Wrap up

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Conclusion

- RMs can provide a good alternative to simulation for generating solution and quality robust schedules bounded by a deadline
- Success of normal approximation, using information about the processing time distribution helps
- Our new measure RM_{16} performs well in both the parallel machine scheduling setting as well as the TUSP setting.

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